Survey Team 1

Research in University Mathematics Education

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ABSTRACT In this report we highlight significant advances in university mathematics education research as well as areas that are in need for additional research insights. We add here to the rich set of literature reviews within the last several years. A novel aspect of this literature review is the fact that the areas of accomplishment and areas for growth were identified based on thematic analysis of survey responses from 119 experts in the field. The review provides a useful overview for both seasoned scholars and those new to research in university mathematics education.

Keywords: University mathematics; Advances; Gaps.

1. Introduction

It is an exciting time for research in university mathematics education (RUME). There are now several major conferences every year across the globe, as well as the fairly new *International Journal of Research in Undergraduate Mathematics Education*, now in its seventh year. With the significant growth in the number of researchers focused on university mathematics education has come the development of research groups and the consolidation of a diverse academic community; RUME is coming to age as a field of research that is beginning to coalesce and develop an identity.

To explore this identity we first surveyed 218 RUME scholars across the world, both well-established scholars and rising stars. We invited these scholars to respond to the following prompt:

What do you see as the most significant advances, changes, and/or gaps in the field of research in university mathematics education? These advances, changes, or gaps might relate to theory, methodology, classroom practices, curricular changes, digital

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environments, purposes and roles of universities, social policies, preparation of university teachers, etc. Please elaborate on just **one or two advances, changes, or gaps** most relevant to your experience and expertise. If possible, please include a few key references.

We received 119 responses. Our next step was to conduct a thematic analysis, which led to the identification of five areas in which there has been considerable progress (advances) and seven areas that are less well-researched (gaps)[8]. Our next step was to conduct a literature review, guided by the identified themes. Perhaps not surprisingly, these two areas are not entirely disjoint.

2. Advances

2.1. Theoretical perspectives

One of the field's major advances considered by several respondents is that we now have a plethora of theoretical perspectives and hence tensions among them can sharpen their constructs and methodologies, and also open the possibility to find commonalities among some of them previously considered to be incompatible. This diversification has contributed to the development of new methods, research topics, and the development and research on theory-based teaching experiences. In particular, the growth of the networking of theories is a recent advance that has added power and depth for analyzing complex learning and teaching phenomenon (Bikner-Ahsbahs and Prediger, 2014; Prediger, et al., 2008)

Related to the networking of theories, recent years have seen the emergence of an interdisciplinary group of scholars interested in using a variety of approaches (logical, cognitive, historical, philosophical, etc.) to address questions which have always been of interest to RUME. This also has increased connections with other disciplines in mathematics and science education and in funding agencies supporting interdisciplinary projects.

Another theoretical advance that is of growing interest is the use of theories that enable insights into issues of equity and social justice. Adiredja and Andrews-Larson (2017) lay out a research agenda for this emerging domain that speaks to the interrelatedness of knowledge, identity, power, and social discourses. While there is still much research that is needed here, we see this new direction as an important advance for the field of university mathematics education research.

2.2. Instructional practices

The research of instructional practices at university level is a rapidly developing area of research. Much of the research on this topic relates to active or inquiry based mathematics education (Artigue, M. & Blomhøj, 2013; Laursen & Rasmussen, 2019).

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Given the myriad calls for instructional reform in university mathematics classrooms, researchers and educators have challenged traditional lecture-based instruction by conducting studies that have provided evidence for the positive effects of innovative student-centered instructions on students' cognitive and affective development.

Active learning, broadly defined as classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking, has repeatedly shown to improve student success and to reduce the equity gap for women and underrepresented students (Freeman et al., 2014; Laursen et al., 2014; Theobald et al., 2020). For example, Freeman et al. conducted a meta-analysis of 225 studies that compared student success in traditional lecture versus active learning in postsecondary science, technology, engineering, and mathematics (STEM) courses and found that average examination scores improved by about 6% in active learning sections, and that students in classes with traditional lecturing were 1.5 times more likely to fail than were students in classes with active learning and the effectiveness of active learning was found across all class sizes. On the other hand, RUME has only begun to deeply explore the culture, experiences, and gendered/racialized interactions in these classes - and how those social factors may be mitigating the students' opportunities to learn (Johnson et al., 2020).

2.3. Professional development of university teachers

Just over a decade ago, Speer et al. (2010) described tertiary level professional development as virtually non-existent and an unexamined practice. Since then there has been considerable progress in this area. In their review of recent research, Winsløw et al., (2021) characterise the literature as comprising primarily small-scale studies of lecturer preparation for teaching. Examples include the effect of backgrounds on teaching (Hernandes-Gomes & González-Martín 2016; Mathieu-Soucy et al., 2018) and the knowledge used in teaching (Musgrave & Carlson, 2017). An inquiry-based experience can be found in (Florensa et al., 2017).

Another advance is the growing collaborative research that builds links between mathematics educators and university mathematicians, links that can increase the pedagogical awareness of the latter group through reflection on teaching practice and provide pedagogical tools (Bardini et al., 2021; Nardi, 2016). However, in order for mathematics educators and mathematicians to collaborate on professional development initiatives, the two groups need to build mutual understanding and trust. Some positive and productive examples of such collaborations are detailed in Jaworksi's (2020) overview of the professional development of university mathematics teachers.

Another very successful collaboration between a group of mathematics educators and mathematicians is detailed in Barton et al. (2014). This collaboration sought to open a two-way channel of communication between the two groups, with the aim to close the ideological perspective gap (Thomas, in press) by understanding the other group's thinking. In this manner, the theoretical and pedagogical knowledge of mathematics education could be conveyed to mathematicians in a manner that focused on their orientations and goals (Schoenfeld, 2010). In turn the mathematician's focus on the crucial elements of mathematics and its learning, along with the role of rigour, was conveyed to the mathematics educators.

2.4. Digital technology

As the review by Clark-Wilson et al. (2020, p. 1223) notes, "in the last two decades the range of digital technology (DT) available has expanded considerably and their facilities and power have also greatly increased. In the light of these changes, the research focus of many has moved from how computers can help with learning to how teachers can make practical use of different types of digital technology to provide students with activities that will enhance their mathematical learning." In addition to new digital technologies, new theoretical perspectives may be applied to the study of technology use at the tertiary level. There have been advances in this area that have emerged. For example, the notion of *Instrumental Orchestration* that arose to consider the collective knowledge building of teachers and students when a technology is appropriated for some mathematical pedagogical purpose (Trouche & Drijvers, 2010). This has recently been developed into the *Documentational Approach to Didactics* (Trouche et al., 2020). The latter combines theoretical elements related to the (instrumental) use of technology, resources, curriculum design, and teachers' professional learning and development to document meaning resource use in a given context with a pedagogical intention.

There have been many advances in terms of DT tools in recent years, such as an ever-expanding internet, clickers, pen-enabled tablets, powerful mobile technology, and interactive retinal screens on smartphones. Several recent studies (e.g., Loch et al., 2014; Maclaren et al., 2018) have demonstrated a very positive feedback from the students on the use of pen-enabled tablets in teaching mathematics, in particular showing a strong preference for this delivery mode compared to other delivery modes.

2.5. Service-courses in university mathematics education

Service-Courses in mathematics are courses provided by mathematicians for students who study engineering, natural sciences, economics, social sciences, psychology, medicine or life sciences, etc. The importance of mathematics for university education is reflected in institutionalized discipline specific mathematics working groups. For example, the European Society for Engineering Education has developed a competence orientated framework for mathematics curricula in engineering education. Research on mathematical service courses also plays an increasing prominent role at national and international conferences in university mathematics education.

Workplace studies figuring out the specific relevance of mathematics for vocational demands have so far mainly considered engineering. For example, considering structural engineers, Gainsburg (2007) has shown that reflections on mathematical concepts become mainly important and more explicit in situations where usual routine procedures do not lead to sufficient results. For this kind of situation Kent and Noss (2003) coined the notion of "breakdown situations".

There are many endeavors making service courses more helpful and relevant to students by implementing or strengthen the discipline related perspective, which intend both to improve the motivation for learning mathematics and the ability to transfer mathematics to discipline (mostly engineering) contexts (Czocher, 2017; Schmidt & Winsløw, 2021). To realize a better connection among service mathematics and other disciplines, a collaboration of mathematics lecturers and lecturers from the other discipline seems to be crucial. Jaworski and Matthews (2011) demonstrate how such a collaboration could be achieved where the lecturers involved conjointly design materials and plan teaching activities. Also related to an inquiry based perspective is the recently started design of so called study-and-research-paths (see for example Barquero et al., 2020) intending to support students in the integration and validation of mathematical practices from different institutional settings.

3. Gaps

3.1. Theories and methods

The development of novel research questions can contribute to the exploration of cogent theories of teaching and learning and the development of sound and innovative methodologies to answer them. As for existing coherent theories there may be a need to develop and extend them further to capture the complexities of the studied phenomena. Studies aiming at developing theory or to test and revise theories using empirical data, and their corresponding methodologies can help in making existing theories more robust and move the field forward. The lack of a shared discourse on meta-level learning is also reflected in the abundance of conceptual and theoretical frameworks suggested by the research literature, which are not necessarily compatible or even commensurable. Developing a shared and explicit discourse for the informal/meta-level content of university mathematics education is a crucial component in any effort to improve pedagogy at this level. While the networking of theories has made some progress on this front, more is needed.

There is in particular a need for frameworks for conducting evaluation research in realistic university settings for testing innovations integrating insights from broader educational research and from university mathematics teachers. Moreover, small-scale qualitative studies are still predominant (Artigue, 2021) or typically only one cycle of research is reported whereas reliability is often associated with multiple rounds of principled research. Also, knowledge of the process of scaling research-based innovations, including effective ways to navigate the political obstacles to shifting undergraduate mathematics courses to be more meaningful, coherent and mathematically engaging for students, is needed.

3.2. Linking research and practice

Some research results have been introduced into university mathematics curriculum, particularly through widely available textbooks for introductory courses. This is, however, not enough. Research results need to inform the teaching of all the mathematics courses through their introduction in the curriculum to significantly improve teaching practices. An important gap that impedes reaching this goal is the lack of research results in advanced university mathematics courses and in the possibility to introduce advanced mathematical ideas in introductory courses. This kind of research can illuminate new ways to motivate students through the teaching of both the usefulness and beauty of mathematics using interesting examples related to students' areas of interest, using what has been found in research about modelling and applications or by introducing new theory-based didactical approaches such as "training using challenges" and paradigms such as "questioning the world" (Chevallard, 2015).

Mathematics teachers and mathematicians need, on the one hand, to have access to different means of communication that go further than research published papers. Mathematicians often find it difficult to read and make sense of the theoretical points of view and the vocabulary used by researchers in mathematics education. Research results need to be made accessible to these other communities, and evidence based instructional practices in ways that are convincing to them. To make this possible not only through professional development programs but other types of media must be used and it is important to offer instructors and mathematicians incentives that can be financial or cultural together with longitudinal support through coaching and mentoring. These incentives and initiatives should clearly be to transform the university culture, which means researchers, teachers and students' transformation, and to support improvements in education. This change is considered fundamental because it is about the culture of mathematics departments, but we know very little about how to affect change (see Reinholz et al., (2019) for a research agenda centered on institutional change).

3.3. Professional development of university teachers

As reviewed in the previous section, there has been some progress in the professional development of university teachers, but much more is still needed. Research could consider the professional needs that university mathematics teachers have, in addition to completing a PhD and possibly a short course in general pedagogy (Winsløw, et al., 2021). How can university mathematics research be developed to contribute to filling those needs, and what measures are needed to engage university mathematics teachers in doing or learning from such research?

The development of programs to achieve strong teaching competence, along with study of their implementation, analysis and improvement comprise an important area for research in university mathematics education. While there have been a number of small studies in this area, the question of how to develop these further and extend them to scale is a topic for research. As Winsløw et al. note, there is a lack of large-scale international studies on teaching practice and its development in university mathematics teacher education.

One aspect to consider in the design of professional development programs is whether there is a role for technology in its design and implementation. For example, could use be made of recent online courses, such as MOOCs designed for in-service mathematics teachers where the aim has been to increase teachers' professional competencies and improve their practices? One advantage of this is the use of online mathematical communication in an open forum. Exchanges from such online mathematical forums have found fruitful use in similar research (Kontorovich, 2018).

There is still room for investigation of the orientations (Schoenfeld, 2010) of mathematics lecturers with respect to their pedagogy. For example, in what manner is mathematics taught by teachers and what do they believe about students who do not do as well as expected in their courses? Do they develop a "deficiency discourse" about their students (e.g., they can't do mathematics because of a lack of ability, are unprepared, unmotivated)? Is their practice influenced by their beliefs about students and is this discourse in any way related to their positive professional identity?

Another issue related to institutional factors that needs attention is to the culture of mathematics teaching and learning at tertiary institutions and how and why this culture continues to result in under-representative participation levels of women and historically marginalized groups (such as those from indigenous cultures and persons of color), particularly at higher levels, where retention rates are extremely low.

3.4. Digital technology

In many schools around the world there has been increased use of digital technology in mathematics learning. Discussing the transition from school to university, Gueudet (2008, p. 252) noted "the question of the effective and possible uses of technology in the secondary–tertiary transition has not been researched yet, as far as I know. Are the abilities with technology built at secondary school exploited at university?" (p. 252). Thus the possible disjoint between school and tertiary use of technology means that issues related to school-university transition may be in need of further research. One interesting question could be, how does a shift from a technological to a non-technological environment influence students' perceptions/interest/attitudes about/for/towards mathematics?

There is now a wealth of sources of mathematical information available to students, who have almost instant access to them, both at home and at their institution. There has been some research related to how they use these sources, such as how engineering students make use of mobile devices and the Internet (Puga & Aguilar, 2015), however, serious questions remain about how and what tertiary mathematics students access and the factors that influence and shape their help-seeking behaviors in the digital era. Examples include research that would systematically analyze how university students use the Internet and mobile devices as a source of mathematical help: What sites do they consult? Why do they consult them? Do they use online real-time support? What makes students trust or prefer one source of information over another one? And how are their mathematical reasoning processes affected by immersing themselves in the use of these digital resources?

Finally, due to the pandemic, courses have increasingly been converting to an online or hybrid format, and when this is pedagogically desirable or not is a topic for research. The research on DT as it relates to the pandemic is only just beginning, detailing both what the field has learned and what open questions remain. We trust that the next ICME survey on RUME will provide substantive insights on this topic.

3.5. Curriculum

Curriculum is an entity present but rarely taken as a unit of analysis in research in university mathematics education. Curricular questions are obviously at the center of all study and examination regulations of mathematics degree programs. However, their treatment rarely relies on specific research, except some empirical studies about students' difficulties or perceptions that are rarely published. At the same time, and at least in the case of mathematics undergraduate degrees, topics, contents, and the structure of study, programs appear surprisingly stable. In the case of mathematics subjects in other types of degrees (engineering, natural sciences, economics and business, etc.), the situation seems more evolving thanks to the introduction of new technologies (especially in the case of statistics subjects), but it is still stable in calculus and linear algebra subjects.

A comparative description of curricula across universities and countries, as well as the processes of external didactic transposition, i.e. the process of selecting and transforming scholarly knowledge into knowledge to be taught, have surprisingly seen little systematic investigation. This is possibly also due to the fact that in mathematics degree programs the knowledge to be taught is rarely questioned. According to (Bosch et al., 2021) for the case of mathematics degrees, there are some differences, for example, between Canada and the USA on the one hand, and Europe on the other, or between types of higher education institutions, such as classical universities and universities of applied sciences. Nevertheless, within each type, the knowledge to be taught has not strongly evolved. This also applies to the external framework conditions, as well as the various dynamics of decision-making processes for changing the curricula or processes of maintaining curricular orders.

3.6. Higher years

Didactic research on learning and teaching in advanced mathematics studies is significantly under-represented. The focus has so far been clearly on the transition from school to university and in the first year of study. This reflects the relevance of the transition and study entry problem, e.g. with regard to dropout rates.

Historically, Felix Klein should of course be mentioned here. Core parts of his "Elementary Mathematics from a Higher Standpoint" actually refer to mathematics that many of today's student teachers do not even get to know in their academic studies. This applies, for example, to knowledge of Fourier analysis that goes beyond the basics, but especially also to knowledge of function theory, e.g. Riemann surfaces and value assignment theorems. Even when students hear about function theory, for example, they usually do not get as far as understanding what Felix Klein considered appropriate knowledge for prospective teachers more than a century ago. Klein considered this knowledge appropriate because it explains why, for example, certain elementary operations have to be restricted in certain ways for mathematical reasons (and not just for didactic reasons of reduction!), and related curricular decisions.

3.7. Interdisciplinarity

Survey responses addressed gaps on many different levels. The most general level concerns research itself. On the one hand, this involves cooperation with mathematicians, engineers, economists, psychologists, etc. For many years, there have been many different kinds of cooperation, for example, agreements between faculties with regard to teaching. What does not seem to exist so far is, among other things, systematic research on these cooperations. What are the benefits of these? How do they take shape? How do they function? Possibilities, limits, etc.? On the other hand, it is about cooperation with researchers from other disciplines in our own research in the narrower sense, i.e. besides mathematicians, with psychologists, university teachers, pedagogues, sociologists, political scientists, historians, anthropologists. In the context of empirical research into learning processes, cooperation with psychologists and educationalists has now been established in many places and, with a view to professionalizing teaching, also with university didacticians. And of course there are also isolated cooperations with other academics. What seems to be missing, however, is a more systematic description and conceptualization of the links. This could be formulated as goals.

The relationship of mathematics to other sciences or the use of mathematics in other sciences also is an area that needs to be addressed. There are several places, such as philosophy or the history of science, in which such connections are examined and the question of what distinguishes mathematics itself and its respective role in other sciences is explored. Research on this is dependent on the respective ideological assumptions and accordingly there are no unambiguous and generally accepted answers here in depth. From the point of view of didactics, however, clarifications in this regard could certainly be regarded as desirable, since they would be of great help in answering the question with which goals, which and how mathematicians, but especially engineers, economists, psychologists, etc., are to be taught.

Last but not least, although in a slightly different way, this also concerns mathematics in itself. It, too, changes its inherent orientation and, to some extent, its character over time. New fields are emerging, such as Big Data and Data Science. Correspondingly, there are new fields of application in other sciences, such as discrete mathematics in electrical engineering, numerical methods also in psychology, etc. This leads directly to questions of teaching: the question of what should be taught in service courses and how is manifold but certainly not sufficient.

4. Conclusion

Grounded in the responses from our RUME scholar survey, we identified five areas in which the field has made significant progress (Theoretical Perspectives, Instructional Practices, Professional Development of University Teachers, Digital Technology, and Service-Courses in University Mathematics Education) and seven areas are in need or further development (Theories and Methods, Linking Research and Practice, Professional Development of University Teachers, Digital Technology, Curriculum, Higher Years, and Interdisciplinarity). These gap areas represent exciting opportunities for the mathematics education research community to conduct scholarly work and help advance the field at large. So while there is now much research-based wisdom, there are also exciting opportunities for new research.

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